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LABOR AND EMPLOYMENT

Forecasting Student Admissions, Graduations, and Numbers in Institutions of Vocational Education

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Abstract—This article presents a mathematical model for long-term high-precision forecasting based on statistical data on student admissions, graduations, and numbers in institutions of vocational education for the period until 2020.

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Forecasting student admissions, student body, and graduations at institutions of vocational education is a vital task. Graduates of the vocational education system represent a major source of labor resources for the job market, and their quantitative forecast assessment is fundamental for the calculation of the prospective workforce balance. Furthermore, graduates of the vocational education system largely contribute to admissions at other levels of vocational education, which has to be taken into account in their assessment. For example, graduates of primary vocational education (PVE) institutions of the current year and previous periods represent 8% of admissions to secondary vocational education (SVE) institutions, and SVE graduates account for 28% of admissions to higher vocational education (HVE) institutions. Thus, a proper assessment of admissions requires the calculation of graduations from the institutions of the vocational education system.

Forecasting student numbers allows one to make long-term assessments of public funding required for education; since 2011, it is calculated per capita for institutions of HVE.

Concept model. Earlier developed models [1–3] considered the numbers of graduates from institutions of vocational education as a linear function of students who were admitted late. Meanwhile, it was assumed that the duration of study for all students admitted to educational institutions was the same, irrespective of the mode and area of study. This period was two years for PVE, four years for SVE, and five years for HVE. This approach only relied on admission and graduation data, which made it easy for use but complicated exact assessments of student numbers. For example, in [4], there was introduced a model describing admissions to institutions of higher vocational education, based on school graduate numbers in the current and previous years. This model did not take into account graduations from and admissions to PVE and SVE institutions. Admissions to SVE and HVE largely depended on the 11th grade graduations in the preceding years (in 2010, the respective share amounted to 15.7% for admissions to HVE and 14.2% for SVE).

In [5], it is suggested to assess the numbers of current-year applicants with secondary (full) general education obtained in the previous years on the basis of data on 11th graders, who were not admitted to vocational education institutions during the graduation year. These data were collected for the three preceding years. However, this model did not provide any quantitative estimates to prove its relevance. The experimental testing of the model did not bring any acceptable results.

Analyzing changes in student numbers of vocational education institutions by years and courses lie at the core of the developed model. In this case, the student numbers in the *j* course and *i* year are determined in respect of the student numbers in the *j*-1 course and *i*-1 year. The graduations in the *i* year are defined in respect of the student body enrolled in senior courses in the *i*-1 year. In particular, this approach has allowed one to assess the number of current-year 11th graders, who have not been enrolled over the last three years, as well as students expelled from the junior courses of SVE and HVE institutions, who can enroll again.

To describe the distribution of flows by admissions to institutions of vocational education, a model, which is similar to the mass transfer model, can be used. The enrollment flows are recorded as balance equations, based on the law of conservation of numbers. This model displays an additive property, and allows one to describe with sufficient precision the collective behavior of enrolling students.

For building a model describing the functioning of the vocational education systems, the following basic assumptions are made:

(a) Eleventh-year graders and holders of secondary (full) general education are entitled to three alternative pathways of vocational education: institutions of

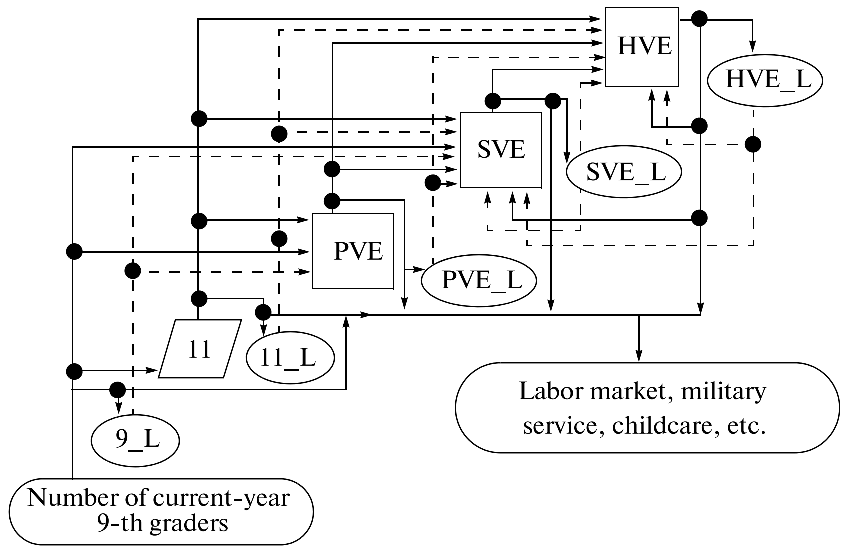

Fig. 1. Advancement of students by levels of vocational education:
PVE, SVE, HVE are educational institutions; 9, 11 are the numbers of current-year 9th and 11th graders; 9_L, 11_L are the numbers of past year graders with the respective level of education: PVE_L, SVE_L, and HVE_L; graduate flows in the current (———) and past (– – –) years, respectively; ☐ the external environment relative to the education system.

PVE with the duration of study between one and two years; institutions of SVE with the duration of study between three and six years; and institutions of HVE with the duration of study between four and seven years.

(b) Ninth-year graders and holders of basic general education are entitled to two alternative pathways of vocational education: institutions of PVE with the duration of study between two and four years and institutions of SVE with the duration of study between four and six years.

(c) Those wishing to study cannot simultaneously enroll in two or more educational institutions.

(d) Students of one educational institution cannot simultaneously study at two or more faculties.

The following features reflect the novelty of the developed model:

1. According to statistical data on admissions, holders of the following educational levels enroll in vocational educational institutions:

– Institutions of PVE require basic general education or secondary (full) general education acquired in the current or previous years. In addition, those without basic general education can enroll in education institutions of PVE.

– Education institutions of SVE require basic general education, secondary (full) general education obtained in the current or previous years, PVE obtained in the current or previous years, or SVE or HVE without specifying the year of graduation.

– Education institutions of HVE require basic general education, PVE, secondary (full) general education acquired in the current or previous years, or HVE without specifying the year of graduation.

2. To forecast the changes in the number of students from one course to another, the following facts need to be taken into account:

– Institutions of SVE admit 9th graders to the first course of study and 11th graders to the second course of study; students enrolling in part-time departments of SVE are admitted to the third course.

– Education institutions of HVE only admit students of all modes of study to the first course.

– Student advancement from one course to another is characterized by the dropout rate determined by statistical data for the previous years.

– Students expelled from the first course are entitled to new enrollments as holders of secondary (full) general education acquired in the previous years.

Structural model. Figure 1 shows a block diagram providing a general description of the possible pathways of vocational education. The interaction between the education system and the external environment is ensured by the inflow of 9th graders into the system (entrance into the system) and outflow of graduate and expelled students from the system, who do not return to studies (as they enter the job market, carry out military service, childcare, etc.).

Figure 2 shows a detailed block scheme of a model describing the functioning of the vocational education system, where the circulation of students is depicted by levels of vocational education, as well as by courses of educational institutions (PVE, SVE, and HVE). In addition to the advancement from one course to another, the circulation of expelled students is also

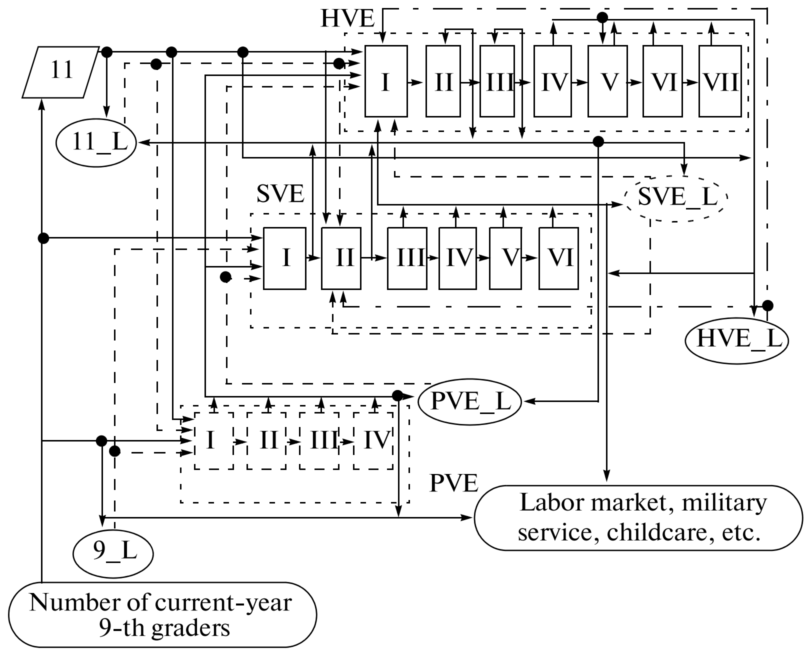

Fig. 2. Block diagram of a model describing the circulation of students in the vocational education system: I–VII corresponding courses; ——— circulation of current year graders; – – – circulation of past years graders; ·— - — mixed flow of graders from institutions of HVE in the current and past years.

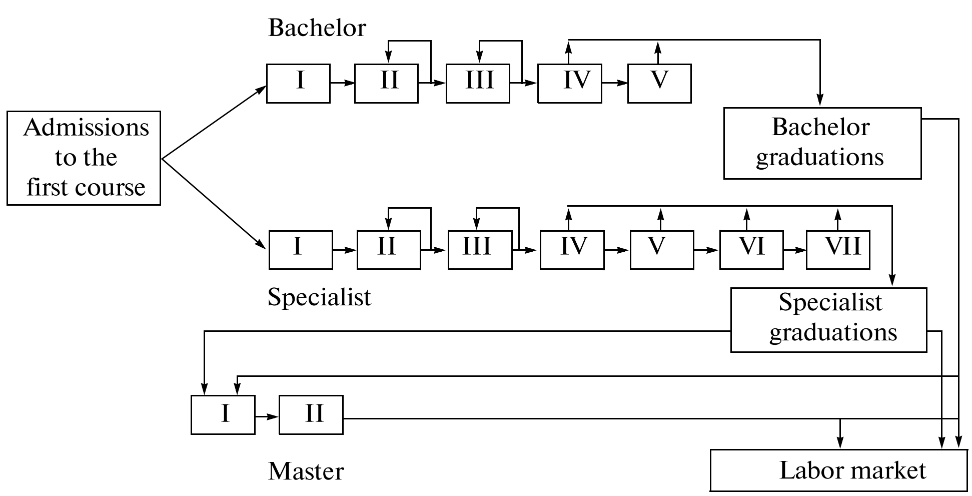

Fig. 3. Model describing the system of HVE, taking into account the level-based training "bachelor–specialist–master."

shown. Presumably, in the subsequent years, students expelled from the first three courses of HVE and from the first two courses of SVE are entitled to new enrollment as graduates of the previous years.

The system of HVE requires a separate and more detailed analysis because of the internal training at the "bachelor, specialist, and master" levels. Figure 3 shows a block diagram of a model describing the system of HVE. According to statistical data, training at the bachelor's level can be between four and five years (for a distant learning mode of study). Training of specialists requires from four to seven years. The duration of study depends on the selected specialty, mode of study (full-time, part-time), and the type of study program (standard, reduced). The duration of study at the master's level is two years.

Such a detailed description of the vocational education system is due to the availability of corresponding data in statistical reports: 1-proftech, 3-proftech, 2-NK, 3-NK, SVE-1, and HVE-1 [6–11].

Mathematical model. A mathematical model is a formalized structural model, which is depicted in Figs. 1–3 in the form of a block diagram.

It is advisable to present the system of balance equations in the form of differential equations, taking into account the discrete time.

The first part of the model describes the distribution of graduates by admissions to institutions of vocational education. The mathematical model is recorded as a system of five differential equations corresponding to the five levels of vocational education (primary, secondary, higher (bachelor), higher (specialist), and higher (master)):

$$\begin{cases} P_i^N = U_i^9 k_i^{9\_N} + U_i^{11} k_i^{11\_N} + U_i^{9r} k_i^{9r\_N} + U_i^{11r} k_i^{11r\_N} \\ + U_i^9 k_i^{9w} \\ P_i^S = U_i^9 k_i^{9\_S} + U_i^{11} k_i^{11\_S} + U_i^{9r} k_i^{9r\_S} + U_i^{11r} k_i^{11r\_S} \\ + V_i^N k_i^{N\_S} + V_{i-1}^N k_i^{Nr\_S} + (V_i^V + V_i^S) k_i^{CSV\_S} \\ P_i^{Vs} = k_i^{Vs} (U_i^{11} k_i^{11\_V} + U_i^{11r} k_i^{11r\_V} + V_i^N k_i^{N\_V} + V_i^{Nr} \\ \times k_i^{Nr\_V} + V_i^S k_i^{S\_V} + V_i^{Sr} k_i^{Sr\_V} + V_{i-1}^V k_i^{V\_V}) \\ P_i^{Vb} = (1 - k_i^{Vs})(U_i^{11} k_i^{11\_V} + U_i^{11r} k_i^{11r\_V} + V_i^N k_i^{N\_V} \\ + V_i^{Nr} k_i^{Nr\_V} + V_i^S k_i^{S\_V} + V_i^{Sr} k_i^{Sr\_V} + V_{i-1}^V k_i^{V\_V}) \\ P_i^{Vm} = k_i^{b\_m} V_i^{Vb} \end{cases} \quad (1)$$

where P^N, P^S, P^{Vs}, P^{Vb}, and P^{Vm} are the admissions to PVE, SVE, HVE (specialist) respectively, HVE (bachelor), and HVE (master); U^9, $U^{11}$are the numbers of 9th and 11th graders; U^{9r}, U^{11r} are the 9th and 11th graders of the previous years enrolling in the current year, respectively; V^N, V^S are the PVE and SVE graduations in the current year; V^{Nr}, V^{Sr} are the PVE and SVE graduations in the previous year; and V^V is the HVE graduations in the current year.

The time-dependent coefficients of model (1) serve for setting (describing) the structure of distribution of graduates by admissions, which changes over time:

$k_i^{9\_N}$, $k_i^{9\_S}$ are the shares of 9th graders of the current year, enrolling in education institutions of PVE and SVE, respectively;

$k_i^{11\_N}$, $k_i^{11\_S}$, $k_i^{11\_V}$ are the shares of 11th graders of the current year, enrolling in education institutions of PVE, SVE, and HVE respectively;

$k_i^{9r\_N}$, $k_i^{9r\_S}$ are the shares of 9th graders of the previous years, enrolling in education institutions of PVE and SVE, respectively;

$k_i^{11r\_N}$, $k_i^{11r\_S}$, $k_i^{11r\_V}$ are the shares of 9th graders of the current year, enrolling in education institutions of PVE, SVE, and HVE, respectively;

k_i^{9w} is a coefficient describing the share of students without basic general education enrolling in education institutions of PVE;

$k_i^{N\_S}$, $k_i^{N\_V}$ are the shares of current-year graduates from education institutions of PVE, enrolling in education institutions of SVE and HVE;

$k_i^{Nr\_S}$, $k_i^{Nr\_V}$ are the shares of current-year graduates from institutions of PVE, enrolling in education institutions of SVE and HVE;

$k_i^{S\_V}$ is the shares of current-year graduates from institutions of SVE, enrolling in education institutions of HVE;

$k_i^{SV\_S}$ is the share of current-year graduates from education institutions of SVE and HVE, enrolling in education institutions of SVE;

$k_i^{Sr\_V}$ is the share of the previous years' graduates from SVE institutions, enrolling in education institutions of HVE in the current year;

$k_i^{V\_V}$ is the share of graduates from education institutions of HVE, enrolling in education institutions of HVE;

k_i^{Vs} is the share of specialists in the admissions to the first courses of institutions of HVE; correspondingly, $1 - k_i^{Vs}$ is the share of students with bachelor's degrees;

$k_i^{b\_m}$ is the share of students with bachelor's degrees, continuing their education towards master's degrees.

To denote the model time, the index $i = i_s, \dots, i_f$ is introduced, which varies from the initial $i_s = 1$ to the final $i_f = 19$ boundaries of the interval (with a step of study in one year). The calendar time (years) of data measurement varies according to the law $t(i) = i + 2001$. Thus, $t(i_s) = 2002$ and $t(i_f) = 2020$.

In the system of equations describing admissions to educational institutions (1), one takes into account the influence of student graduations from these institutions. In this way, a positive correlation is established between admissions and graduations.

The second part of the model describes the dynamics of graduations from institutions of vocational education. Graduations from SVE and HVE institutions depend on the numbers of students in senior courses in the previous year. The corresponding equations are recorded as follows:

$$\begin{cases} V_i^S = QV^S (ktgS_{i-1}^3 + ktgS_{i-1}^4 + ktgS_{i-1}^5 + ktgS_{i-1}^6) \\ V_i^V = V_i^{Vs} + V_i^{Vb} + V_i^{Vm} \\ V_i^{Vs} = QV^{Vs} (ktgVs_{i-1}^4 + ktgVs_{i-1}^5 + ktgVs_{i-1}^6 + ktgVs_{i-1}^7), \\ V_i^{Vb} = QV^{Vb} (ktgVb_{i-1}^4 + ktgVb_{i-1}^5) \\ V_i^{Vm} = QV^{Vm} ktgVm_{i-1}^2 \end{cases} \quad (2)$$

where V_i^S is the number of graduations from SVE institutions; QV^S is a coefficient characterizing SVE graduations; $ktgS_i^j$ is the number of SVE students in the i year in the j course; V_i^V is the total number of HVE graduations, V_i^{Vs} is the number of specialist graduations; V_i^{Vb} is the number of graduations with bachelor's degrees; V_i^{Vm} is the number of graduations with master's degrees; QV^{Vs}, QV^{Vb}, QV^{Vm} are coefficients characterizing HVE graduations with specialist, bachelor's, and master's degrees, respectively; $ktgVs_i^j$ is the number of HVE specialist students in the i year in the j course; $ktgVb_i^j$ is the number of HVE students with bachelor's degrees in the i year in the j course; and $ktgVm_{i-1}^2$ is the number of second-course master's students in the previous year.

The third part of the model consists of 20 equations of type (3), describing the dynamics in student numbers by courses in institutions of vocational education:

$$ktgO_i^k = qO^{k-1,k} ktgO_{i-1}^{k-1}, (3) \tag{3}$$

where $qO^{k-1,k}$ is a coefficient characterizing the advancement from the k-1 course to k course for the level of education $O \in \{S, Vs, Vb, Vm\}$, (S, SVE; Vs, HVE (specialist); Vb, HVE (bachelor); Vm, HVE (master). The coefficient $qO^{k-1,k}$ reflects the probability of advancement from the k-1 to k course. Respectively, the value $1 - qO^{k-1,k}$ shows the probability of leaving education.

To assess the student body and graduations from institutions of PVE, one uses an approach, which is different from the one for SVE and HVE, because there is no statistical data on student distributions by courses for institutions of PVE. Data on admissions to institutions of PVE are presented from the perspective of the duration of study (from one to four years). On this basis, the maximum possible number of graduations $VN_i^{\max}$ and the numbers of students in PVE institutions $ktgN_i^{\max}$ are calculated. After the obtained values have been compared with factual data on student bodies and graduations, the annual dropout rates for institutions of PVE for student admissions QV^N and the student body $QktgN$ are calculated. Thus, student graduations from and the student body in institutions implementing PVE programs is presented in the following form:

$$V_i^N = QV^N VN_i^{\max}, \tag{4}$$

$$VN_i^{\max} = N_{i-1}^1 + N_{i-2}^2 + N_{i-3}^3 + N_{i-4}^4. \tag{5}$$

Similarly, for the PVE body:

$$ktgN_i = QktgN ktgN_i^{\max}, \tag{6}$$

$$ktgN_i^{\max} = QktgN[N_i^1 + N_i^2 + N_i^3 + N_i^4 + N_{i-1}^2 + N_{i-1}^3 + N_{i-1}^4 + N_{i-2}^3 + N_{i-2}^4 + N_{i-3}^4], \tag{7}$$

where N_i^k is the number of students admitted to PVE institutions in the i year to programs with the duration of study of k years.

Assessment of external impacts. In the above-suggested model, 9th and 11th graders serve as a factor exercising an external influence on the vocational education system. The number of general education school graduates directly influences the admissions to institutions of vocational education. In turn, the number of school graduates directly depends on the demographic birth rate. These patterns are identified and described in [12].

The model described in this article is based on the forecasted numbers of 9th and 11th school graders until 2020, which were obtained in [12] using a linear equation connecting the demographic birth rate with the number of 9th graders as follows:

$$U_i^9 = 0.13X_{i-14} + 0.75X_{i-15}, \tag{8}$$

where U_i^9 is the number of 9th graders and X_{i-14}, X_{i-15} are the numbers of births 14 and 15 years ago, respectively. The 0.13 and 0.75 coefficients are defined by regression analysis. The approximation error during retrospective data testing is 3.9% [12].

The number of 11th graders U_i^{11} is defined on the basis of the number of 11th graders two years before using a linear equation:

$$U_i^{11} = 0.65U_{i-2}^9. \tag{9}$$

The coefficient connecting the numbers of 9th and 11th graders is also defined by regression analysis. The average approximation error for using equation (9) for a postforecast period is less than 2% (1.88%) [12].

Following this logic, one can forecast the number of 9th and 11th graders for the period between 2010 and 2020, based on the annual numbers of births between 1993 and 2003.

Figure 4 shows the modeling results, which were obtained by the authors [12] and used as initial data on the numbers of 9th and 11th graders in the present study.

The assessment of the number of *9th graders of the previous years* is defined on the basis of 9th graders who have not enrolled in further vocational education for the last 3 years. Thus, an assumption is made that the nonenrolled 9th graders of the i year only increase the admissions in the next 3 years: $i+1$, $i+2$, and $i+3$; at the same time, the rest of the graduates from the nonenrolled student body are not taken into account for the year $i+4$.

The use of such a time diapason (lag) is required by the results obtained from the correlation analysis on the dependence between the numbers of graduates who have not been enrolled in educational institution over the past years and the admissions to institutions of vocational education. The strongest correlation between the indicated values can be observed under shifts to 1, 2, or 3 years. The influence of school graduates who graduated more than 3 years ago is insignificant.

In addition, it is important to meet the requirement for the share of 9th graders to become less and less significant in the admissions of each subsequent year. This condition can be met with the help of the coefficients of exponential smoothing, which are characterized by an ability to depend on one parameter and be equal to 1 in the sum:

$$\gamma + \gamma(1-\gamma) + (1-\gamma)^2 = 1. \quad (10)$$

If $\gamma > \frac{1}{2}$, the following property is revealed:

$$\gamma > \gamma(1-\gamma) > (1-\gamma)^2. \quad (11)$$

These two properties take into account the progressively diminishing share of nonenrolled graduates, as well as its fullness; i.e., the numbers of graduates are fully distributed over a period of 3 years. Thus, the sum defining the number of 9th graders of the previous years in the admissions is described by the following equation:

$$U_i^{9r} = \gamma U_{i-1}^9 k_{i-1}^{9r} + \gamma(1-\gamma)U_{i-2}^9 k_{i-2}^{9r} + (1-\gamma)^2 U_{i-3}^9 k_{i-3}^{9r}, \quad (12)$$

where k_i^{9r} is the share of nonenrolled 9th graders in the total number of students in the i year.

The optimal value of the $\gamma^* = 0.68$ coefficient was evaluated by the least squares method:

$$\gamma^* = \arg\min_{0,5<\gamma<1} \left\{ \sum_i \left(U_i^{9r^{fact}} - U_i^{9r}(\gamma) \right)^2 \right\}, \quad (13)$$

where $U_i^{9r^{fact}}$ is the factual numbers of 9th graders of the previous years enrolling in the current year.

Eleventh graders of the past years. An analysis of statistical data shows that as a result of the sharp demographic decline since the mid-1990s, at present the number of 11th school graders decreases every year (Fig. 5).

Meanwhile, the number of current-year 11th graders does not reduce at the same pace in the admissions to higher educational institutions because of the fact that the share of 11th graders enrolling in higher educational institutions after school graduation is increasing (Fig. 6).

Besides, analysis of statistical data on admissions to institutions of vocational education shows that the share of 11th graders of the past years is not decreasing in the admissions to these institutions (in 2006, their number was 194400 or 11.7% of all admission; in 2009, 218900 or 14.2%).

By testing the adequacy of the model describing the participation of 11th graders of the previous years in the admissions of the current year, which is presented in [1, 2], it was found that approach (12) applied for the assessment of 9th graders of the past years in the admissions to institutions of vocational education is not appropriate for the assessment of 11th graders.

As a result, a more complicated solution has been found. The number of enrolling students in the current year, who obtained secondary (full) general education in the previous years, was analyzed separately by edu-

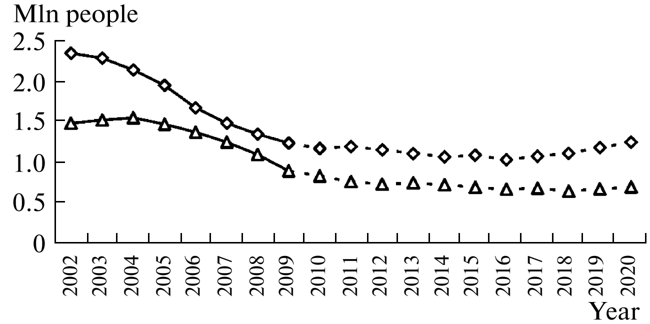

Fig. 4. Dynamics of the numbers of 9th and 11th school graders (the data for 2002–2009 is factual; for 2010–2020, calculated): –◇– 9th year (actual); - -◇- - 9th year (model); –△–11th year (actual); - -△- - 11th year (model).

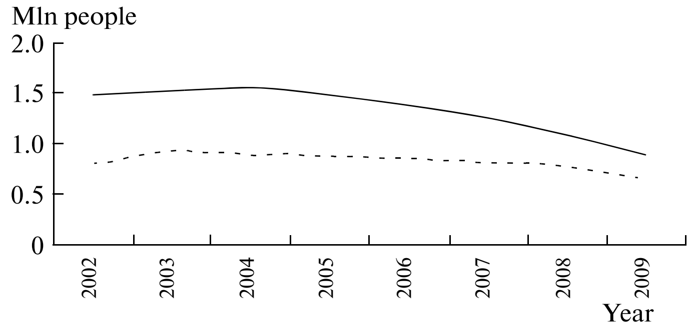

Fig. 5. Dynamics of the number of 11th school graders (——) and students enrolled (- - - -) in institutions of HVE in the current year in the period 2002–2009.

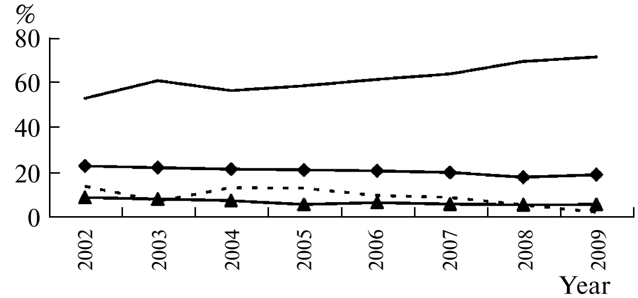

Fig. 6. Dynamics of the distribution of current-year 11th graders by admissions to vocational education institutions in the period 2002–2009: —— EI of HVE; —◆— EI of SVE; —▲— EI of PVE; - - - -not enrolled.

cation types: full-time education and so-called part-time education, including all other types (part-time, extramural, and external). Therefore,

$$U_i^{11r} = U_i^{11r\_O} + U_i^{11r\_NO}, \qquad (14)$$

where $U_i^{11r\_O}$ is the number of holders of secondary (full) general education obtained in the past years, enrolling in full-time education programs in the current year and $U_i^{11r\_NO}$ is the number of people who enroll in part-time programs.

The conducted correlation analysis demonstrates a high dependence (on the Chaddock scale) between the number of students with secondary (full) general education obtained in the past years, admitted in the current year to full-time programs, and the number of 11th graders with a lag of 1,2, and 3 years, who were not admitted to institutions of vocational education.

In this case, the number of students admitted to full-time education programs of vocational education institutions is defined by the following equation, which takes into account the number of 11th graders who have not enroll in education programs over the last 3 years:

$$U_i^{11r\_O} = \alpha_1 k_{i-1}^{11r} U_{i-1}^{11} + \alpha_2 k_{i-2}^{11r} U_{i-2}^{11} + \alpha_3 k_{i-3}^{11r} U_{i-3}^{11}, \qquad (15)$$

where k_i^{11r} is the share of nonenrolled 11th graders in the admission and $0 < \alpha_n < 1$ ($n = 1, 2, 3$) is the share of students enrolling in the next years (in n years) in the number of nonenrolled 11th graders. The α_n coefficients are found by the least squares method: $\alpha_1 = 0.04$, $\alpha_2 = 0.31$, $\alpha_3 = 0.28$, respectively. The average relative error is 10.6%.

The large numbers of the previous years' 11th graders in admissions cannot be explained only by the nonenrolled graduates over the last 3 years. Students, who were expelled from junior courses of educational institutions and undertook further attempts to enroll in the subsequent years, are considered as an additional source of holders of secondary (full) general education obtained in the previous years, in addition to the nonenrolled 11th graders.

The annual number of students expelled from all levels of vocational education $U11^{out}{}_i$ is defined on the basis of student numbers by courses by the following equation:

$$U11_i^{out} = \sum_{j=2}^{4} (ktgVs_{i-1}^{j-1} - ktgVs_i^j)$$
$$+ \sum_{j=2}^{4} (ktgVb_{i-1}^{j-1} - ktgVb_i^j) + \sum_{j=2}^{4} (ktgS_{i-1}^{j-1} - ktgS_i^j) \qquad (16)$$
$$+ (1 - QV^N)(N_{i-1}^1 + N_{i-2}^2 + N_{i-3}^3 + N_{i-4}^4).$$

The admission of 11th graders of the past years to part-time courses consists of expelled students and part of the 11th graders, who have not been enrolled in courses over the last 3 years, distributed, as 9th graders, with the coefficient of exponential smoothing:

$$U_i^{11r\_NO} = \beta U11_i^{out} + (1 - \sum_{n=1}^{3} \alpha_n)(\mu U_{i-1}^{11} k_{i-1}^{11r}$$
$$+ \mu(1-\mu) U_{i-2}^{11} k_{i-2}^{11r} + (1-\mu)^2 U_{i-3}^{11} k_{i-3}^{11r}), \qquad (17)$$

where is the share of students expelled from institutions of vocational education and enrolling again; in this equation, the $1 - \sum_{n=1}^{3} \alpha_n$ multiplier takes into account the enrollment of part of students in full-time education programs. The coefficients calculated with the least squares method $\beta = 0.42$ and $\mu = 0.22$. The average relative error for the 11th graders of the past years who enroll in part-time education programs in the current year is 8.5%.

The average relative error for the sum of students enrolling in full- and part-time education programs of the factual numbers of enrolling 11th graders of the past years is 8.6%.

Thus, obtained model (14–17), which takes into account the 11th graders who have not enrolled in courses over the last 3 years and students expelled from junior courses, allows one to forecast the number of current-year graduates with secondary (full) general education obtained in the past years with a maximum error of 10%.

Identifying the model's parameters. Unlike the above-noted description, the detailed description of model (1–7) contains 34 linear finite differential equations and 53 parameters, 32 of which are constant and 21 are nonstationary.

System equations (1–7) describing the dynamics of student admissions, graduations, and numbers in institutions of vocational education are valid both for public and private institutions of HVE, PI and PrI, correspondingly. Thus, if one considers private higher education institutions, the number of equations increases to 54; the number of stationary parameters, to 46; and the number of nonstationary ones, to 30.

Among constants are the parameters describing the probabilities of students advancing from one course to another, as well as student graduations from senior courses., These parameters are relatively reliably identified as averages of retrospective student data describing the number of students by courses and years.

Nonstationary parameters include those that characterize the distribution of graduates from schools and vocational education institutions by admissions to these institutions, because they reflect the obvious patterns of changes entailed by the objective socioeconomic situation. For identifying such parameters, we used the following approximation by a logistic curve:

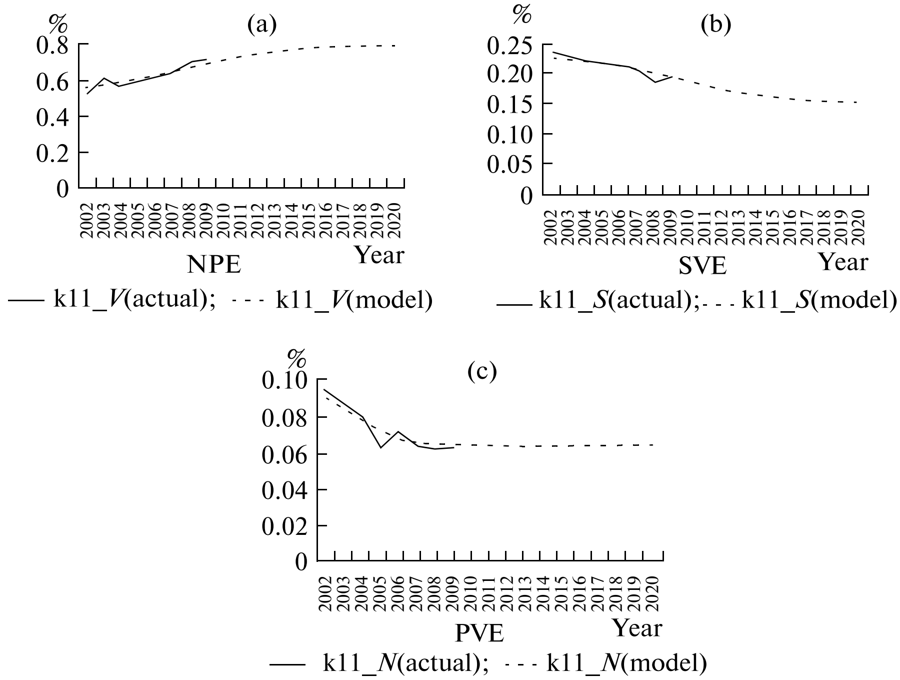

Fig. 7. Share of 11th graders enrolling in educational institutions: Parameters of the logistic curves: (a) a = 0.39, b = 8.46, c = 0.26; (b) a = 0.36, b = 11.48, c = –0.08; (c) a = 0.97, b = 6.77, c = –0.03.

$f(t) = \dfrac{c}{1 + be^{-at}}$. Figure 7 depicts examples of such an approximation for the distribution of current-year 11th graders by admissions to institutions of HVE, SVE, and PVE, taking into account the restrictions on the number of 11th graders:

$$k11\_V + k11\_S + k11\_N \le 1, \qquad (18)$$

where $k11\_V$, $k11\_S$, and $k11\_N$ are the respective shares of 11th graders enrolling in institutions of HVE, SVE, and PVE. The average relative errors are equal to 3.1, 1.9, and 4.5%.

Modeling results. After the forecast, an assessment was conducted for testing the adequacy of the model in order to compare the model and factual data.

Figures 8a–8c depict the model calculation results for some of the observed values: admissions to and graduations from institutions of PVE and SVE, as well as public and private institutions of PVE. The relative average errors were the following: 5.8% for the PVE admissions, 3.9% for the SVE admissions, 4% for the HVE admissions, 3.6% for the PVE graduation, 1.8% for the SVE graduations, and 2.5% for the graduations from public and private institutions of HVE.

For institutions of HVE, the model takes into account training at the "bachelor–specialist–master" levels. Meanwhile, the conditions for a gradual transition to a level-based training system are met, namely, a smooth decrease in admissions to specialist programs and an increase in admissions to bachelor programs. Figures 8d–8f show the model's calculation results for the admissions and graduations of bachelors, specialists, and masters. The average relative errors are 5.4% for the bachelor admissions, 4.0% for the specialist admissions, 6.2% for the bachelor graduations, 2.2% for the specialist graduations, 8.5% for the master admissions, and 4.2% for the master graduations.

In addition, model (1–7) allows one to calculate the student bodies in institutions of vocational education. Figure 9 provides the calculation results. The average relative error for student bodies is 9.1% for PVE, 1.2% for SVE, and 1.9% for HVE.

Therefore, the developed mathematical model allows one to make forecasts for admissions to and graduations from institutions of vocational education, based on forecasts of the numbers of 9th and 11th school graders, as well as on data on the current numbers of students by courses.

This mathematical model involves a system of 34 linear differential equations containing 32 stationary and 21 nonstationary parameters.

This model allows one to make forecasts of student admissions, graduations, and numbers in institutions of PVE, SVE, and HVE with the maximum relative error of 10%. Moreover, regarding the system of higher vocational education, this model allows one to assess student admissions, numbers, and graduations at the bachelor's, specialist, and master's levels in public and private higher education institutions.

According to the calculation results of the developed model, a decline in admissions to all education institutions of vocational education at all levels will

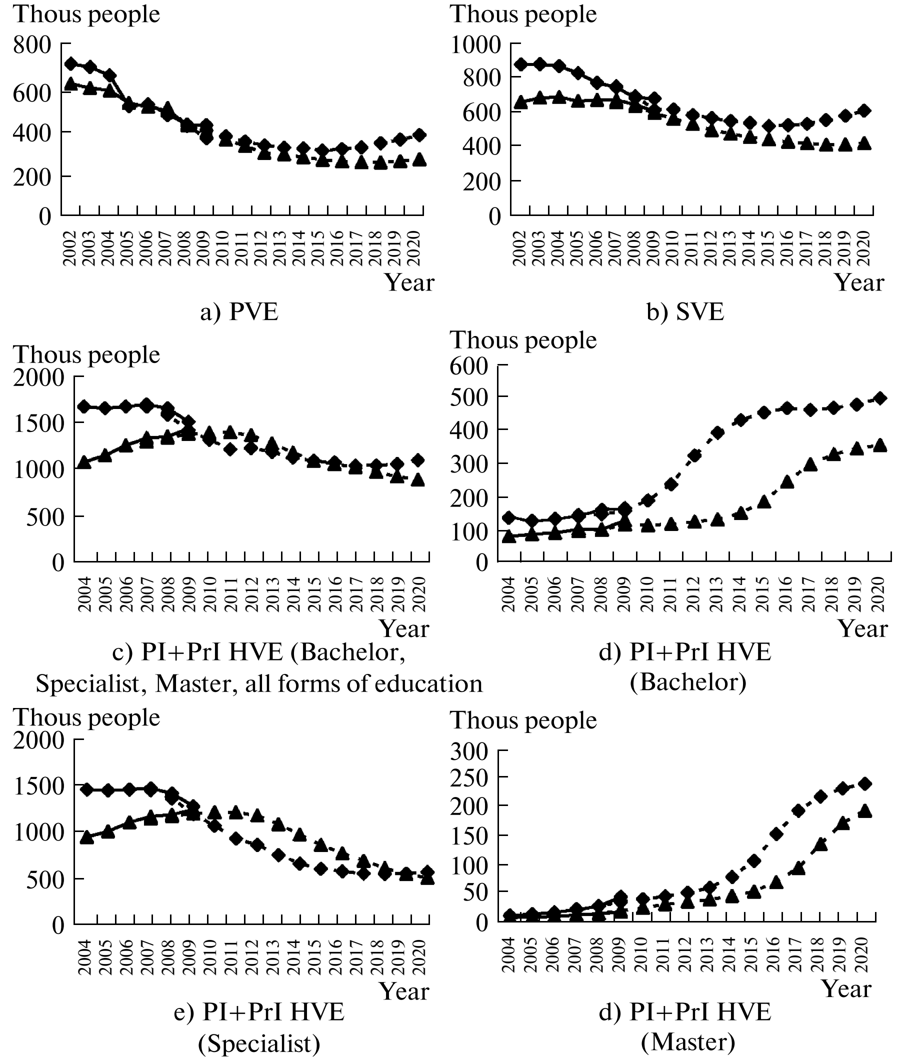

Fig. 8. Calculation results of the model describing admissions to and graduations from educational institutions: —◆— admission (actual); - ◆ - admission (model); —▲— graduation (actual); - ▲ - graduation (model).

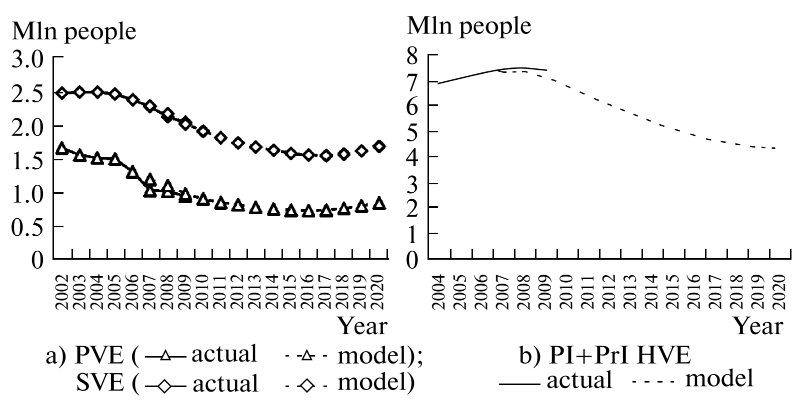

Fig. 9. Calculation results of student numbers.

remain to be observed in the coming years (until 2015). This decline can be explained by the reduced birth rates in the 1990s. Starting from 2015, admissions will demonstrate insignificant growth. As a result of the decline in admissions, the numbers of vocational education graduates will also continue to decrease. Thus, the total number of admitted students in 2010 (2.415 million students) to all levels of education will drop to 1.790 million students in 2015, where it will stabilize and remain practically unchanged until 2020. Such a 25% reduction in the supply of young people to the labor market can result in an increase in the average age of employees.

One should specifically highlight the system of higher vocational education. The model considers the transition of higher education to a three-tier system of training (bachelor's, specialist, and master's degrees). Admission to specialist programs will continue to decrease, while admissions to bachelor's and master's programs will grow. By 2015, the ongoing transformation processes in the admission structure at the bachelor's, specialist, and master's levels will be accomplished according to the gradual transition model. In view of the fact that graduations lag behind admissions, the stabilization of the admission structure will require another 3–4 years. This means that the concept "bachelor", i.e., a specialist with higher vocational education, will become common for the labor market by approximately 2018.

Since 2013, similarly to admissions, the same changes will take place for graduations from the respective levels of higher education. By 2020, the share of specialists in admissions will drop from 87 to 48%, while the shares of people with master's and bachelor's degrees will increase from 3 to 18% and from 10 to 34%, respectively.

All the above-noted can be used as a forecast tool for decision making in the area of vocational education.

With the respective adjustment and availability of initial statistical data, this model can be applied for calculating student admissions, student body, and graduation numbers of education institutions at three levels of vocational education, as well as at the levels of the Russian Federation and its federal subjects.

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